Tilted Cosmology

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Motivation

Observational

- Bulk peculiar motions appear to be the norm rather than the exception.
- No "realistic" observer in the universe seems to follow the CMB frame.
- Typical bulk-flow sizes and speeds are $\gtrsim$100 Mpc and $\gtrsim$100 km/sec.
- Reports of bulk flows in excess of those anticipated by ΛCDM.

Historical

- Relative motions have been known to "contaminate" the observations.
- Relative motions have occasionally led to gross misinterpretations of reality.

Theoretical

- Most theoretical cosmological studies bypass peculiar motions.
- The few and sparse studies of peculiar flows are typically Newtonian.

Bulk peculiar flows and GR

Moving matter "gravitates"

- Peculiar flows are matter in motion.
- Moving matter means nonzero "peculiar" flux.
- In GR energy fluxes gravitate as well.

For example, assuming pressureless matter, the stress-energy tensor reads

$$T_{ab} = \rho u_a u_b + 2q_{(a}u_{b)}\,,$$

where ρ is the density, q_a is the flux and u_a is the observer's 4-velocity.

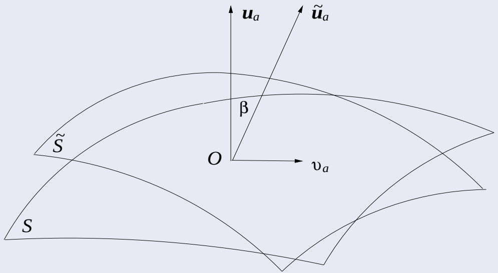
In the presence of peculiar flows

- There is always nonzero "peculiar" flux ($q_a \neq 0$).
- The peculiar flux feeds into the EFE, into the conservation laws, an so on.
- The gravitational input of the peculiar flux has been largely bypassed.

We will consider the implications of the peculiar flux for the evolution of peculiar velocities themselves and for our interpretation of the observational data.

Tilted cosmologies

- Best suited to the study of peculiar motions.
- Employ a perturbed "tilted", almost-EdS universe (for simplicity).
- Use relativistic linear cosmological perturbation theory.
- Introduce "reference" and "tilted" observers, with 4-velocities u_a and $\tilde{u}_a$.



For non-relativistic peculiar velocities (with $v^2 \ll 1$)

$$\tilde{u}_a = u_a + v_a \,. \qquad (1)$$

"Peculiar" flux and "peculiar" 4-acceleration

Linear relations between the two frames

On Friedmann backgrounds and assuming zero pressure,

$$\tilde{\rho} = \rho\,, \qquad \tilde{p} = p = 0\,, \qquad \tilde{\pi}_{ab} = \pi_{ab} = 0 \qquad \textit{and} \qquad \tilde{q}_a = q_a - \rho v_a\,.$$

The linear "peculiar" flux

Adopting the conventional view, the flux vanishes in the "tilted" frame. Then, $\tilde{q}_a = 0$ and

$$q_a = \rho v_a\,,$$

is the linear peculiar flux in the reference frame.

The linear "peculiar" 4-acceleration

The flux input to the (perturbed) EFE feeds to the linear conservations laws

$$\dot{\rho} = -3H\rho - \mathrm{D}^a q_a \qquad \textit{and} \qquad \rho A_a = -\dot{q}_a - 4Hq_a\,,$$

where A_a is the linear peculiar 4-acceleration in the reference frame.

The nonzero flux (q_a) ensures nonzero 4-acceleration (A_a), even in the absence of pressure.

Linear evolution of peculiar velocities

The linear driving force

Substituting $q_a = \rho v_a$ into the momentum conservation law gives

$$\dot{v}_a + Hv_a = -A_a \,, \qquad (2)$$

making the 4-acceleration the driving force of the peculiar-velocity field.

To proceed, we need a linear expression for A_a.

The non-relativistic result

- Newtonian: $A_a \to \partial_a\Phi$, where Φ is the gravitational potential.
- Quasi-Newtonian: $A_a \to \mathrm{D}_a\varphi$, where φ is a scalar potential (with $\dot{\varphi} = -H$).

Both studies bypass (for different reasons) the gravitational input of the flux and both arrive at the same growth-rate for the linear peculiar-velocity field, namely

$$v_a \propto t^{1/3} \,.$$

The quasi-Newtonian approach severely constrains the (perturbed) spacetime, leading to Newtonian-like equations and results (warning comments in Ellis, et al (2012)).

Relativistic linear evolution of peculiar velocities

The relativistic linear 4-acceleration

Taking the spatial gradient of the linear energy-conservation law gives

$$\dot{\Delta}_a = -\mathcal{Z}_a - 3aHA_a - a\mathrm{D}_a\vartheta\,. \tag{3}$$

Also, the gradient of Raychaudhuri's formulae leads to

$$\dot{\mathcal{Z}}_a = -2H\mathcal{Z}_a - \frac{1}{2}\,\rho\Delta_a - \frac{9}{2}\,aH^2A_a + a\mathrm{D}_a\mathrm{D}^bA_b\,, \tag{4}$$

with $\vartheta = \mathrm{D}^a v_a$. Also, Δ_a and $\mathcal{Z}_a$ describe inhomogeneities in the density and the expansion.

The relativistic result

Substituting the 4-acceleration from Eq. (2) and assuming zero spatial curvature, the above expressions combine to the following (non-homogeneous) differential equation

$$\ddot{v}_a + H\dot{v}_a - \frac{3}{2}\,H^2v_a = \frac{1}{3aH}\left(\ddot{\Delta}_a + 2H\dot{\Delta}_a - \frac{3}{2}\,H^2\Delta_a\right)\,.$$

The homogeneous component of this DE provides the "minimum growth" solution

$$v = C_1\,t + C_2t^{-2/3}\,,$$

in accord with the mathematical theory of differential equations.

Relativity and the bulk-flow puzzle

GR can relax the (Newtonian) ΛCDM limits

- Relativity supports stronger growth for linear peculiar velocities.
- This means faster and deeper residual large-scale bulk flows.
- The bulk-flow puzzle can be solved within conventional physics.

More details in: *CGT, Astrophys. J. (2025)*.

The key role of the peculiar flux

In GR peculiar flows "gravitate".

Accounting for the gravitational input of the peculiar flux is what distinguishes the relativistic studies of peculiar motions from the rest.

Bypassing, for one reason or another, the gravitational input of the peculiar flux leads to Newtonian results. Additional examples are:

- The Zeldovich approximation in GR (quasi-Newtonian $\rightarrow$ pancakes (Newtonian)).
- The Meszaros effect in GR (Landau-Lifshitz/energy frame $\rightarrow$ stagnation (Newtonian)).

Cosmic deceptions due to peculiar motions

Motivation

Kinematic illusions due to relative motion

- In everyday life (usually brief and sometimes amusing).
- When interpreting astronomical observations (can be serious and long-lasting).

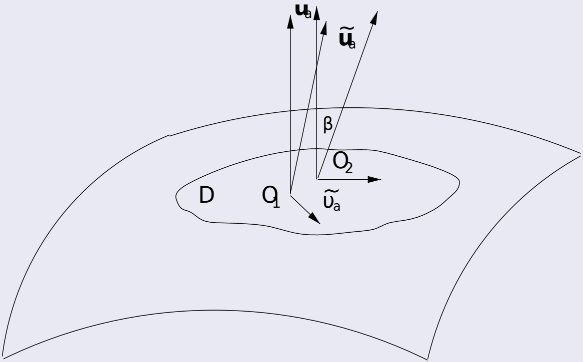
What about the interpretation of cosmological data?

After all, we are moving observers in the universe.

Bulk-flow observers vis-a-vis CMB observers

Consider observers inside a large-scale bulk flow (D), moving with peculiar velocity v_a relative to their CMB partners.

The associated 4-velocities $\tilde{u}_a$ (bulk flow) and u_a (CMB) are related by $\tilde{u}_a = u_a + v_a$.



The scalars $\tilde{\Theta} = \tilde{\mathrm{D}}^a \tilde{u}_a$ and $\Theta = \mathrm{D}^a u_a$ measure the expansion rate in the two frames.

Bulk-flow frame vs CMB frame

The universal kinematics, as measured in the two frames, differ and they are related by

Two expansion scalars

The expansion scalars satisfy the linear relations

$$\tilde{\Theta} = \Theta + \vartheta \qquad \textit{and} \qquad \tilde{\Theta}' = \dot{\Theta} + \dot{\vartheta}\,, \qquad \textit{where} \qquad |\vartheta|/\Theta \ll 1\,,$$

with $\vartheta = \mathrm{D}^a v_a \gtrless 0$ being the local expansion/contraction scalar.

Two deceleration parameters

The deceleration parameters measured in the bulk-flow and the CMB frames are

$$\tilde{q} = -\left(1 + \frac{3\tilde{\Theta}'}{\tilde{\Theta}^2}\right) \qquad \textit{and} \qquad q = -\left(1 + \frac{3\dot{\Theta}}{\Theta^2}\right)\,. \tag{5}$$

Therefore $\tilde{q} \neq q$.

Comparing the deceleration parameters

The "correction/contamination" term

On the EdS background, Eqs. (5a) and (5b) combine to the (relativistic) linear relation

$$\tilde{q} = q + \frac{1}{9}\left(\frac{\lambda_H}{\lambda}\right)^2 \frac{\vartheta}{H}\,, \qquad where \qquad \vartheta \gtrless 0 \qquad and \qquad |\vartheta|/H \ll 1\,, \tag{6}$$

with $\lambda_H = 1/H$ being the Hubble scale and λ the bulk-flow scale.

The above also applies to all FRW and Bianchi backgrounds, unless $\Omega \gg 1$, or $\sigma/H \gg 1$.

The λ and ϑ-dependent peculiar-motion effect

The scale-dependence implies that:

$$\tilde{q} \neq q \quad \text{when} \quad \lambda \ll \lambda_H\,, \quad \text{while} \quad \tilde{q} \to q \quad \text{when} \quad \lambda \gtrsim \lambda_H \quad \text{(expected)}.$$

The ϑ-dependence implies that:

$$\tilde{q} > q \quad \text{when} \quad \vartheta > 0\,, \quad \text{while} \quad \tilde{q} < q \quad \text{when} \quad \vartheta < 0.$$

Mimicking accelerated expansion: Deception No 1?

Contracting bulk flows

In a (slightly) contracting bulk flow with $\vartheta < 0$

$$\tilde{q} = q - \frac{1}{9}\left(\frac{\lambda_H}{\lambda}\right)^2 \frac{|\vartheta|}{H}\,, \qquad \textit{where} \qquad |\vartheta|/H \ll 1\,. \tag{7}$$

On small scales (with $\lambda \ll \lambda_H$) the "correction/contamination" term dominates.

Then, it is possible to have $\tilde{q} < 0$ locally and $q > 0$ globally.

Two kinds of bulk-flow observers

- "Uninformed" observers: Unaware of their bulk flow's local contraction.
- "Informed" observers: Aware of their bulk flow's local contraction.

Two different interpretations

- Uninformed observers: Likely to mistake their local contraction for global acceleration. Like passengers on a slowing train may think that the other trains are accelerating away.
- Informed observers: Know that all could be a mere artefact of their relative-motion.

Mimicking recent acceleration: Deception No 2?

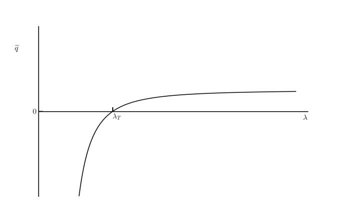
The transition length

The correction/contamination term dominates the right-hand side of (7) at the "transition length"

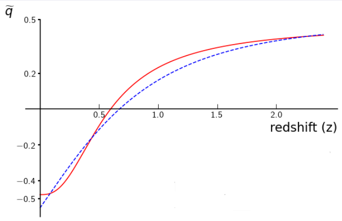
$$\lambda_T = \sqrt{\frac{1}{9q}\frac{|\vartheta|}{H}}\,\lambda_H\,, \qquad \textit{so that} \qquad \tilde{q} = q\left[1 - \left(\frac{\lambda_T}{\lambda}\right)^2\right]. \tag{8}$$

Therefore, $q < 0$ when $\lambda < \lambda_T$.

$\vartheta = \langle\vartheta\rangle =$constant



$\vartheta = \vartheta(\lambda) = \lambda^2/(\alpha + \beta\lambda^3)$



The uninformed observers are prone to think that cosmic acceleration started at λ_T.

Some numbers: Estimating $\tilde{q}$ and λ_T when $\vartheta < 0$

Turning to the bulk-flow surveys

Setting $q = 1/2$, $H_0 \simeq 70\ \mathrm{km/secMpc}$ and $\vartheta = \langle\vartheta\rangle \simeq -\langle v\rangle/\lambda$

Survey	λ (Mpc)	$\langle v\rangle$ (km/s)	$\tilde{q}$	λ_T (Mpc)
Nusser & Davis (2011)	150	250	-2.66	375
Colin, et al (2011)	250	260	-0.24	300
Watkins, et al (2023)	200	395	-1.72	415
Watkins, et al (2023)	250	430	-0.69	375

The relative-motion effect is local, but the affected scales are large enough to make it look like a recent global event.

Conflicting theoretical interpretations

Although all the bulk-flow observers measure the same deceleration parameter, they interpret their observation differently.

Uninformed observer's approach

They typically add an effective pressure term (p_{ef}) to Raychaudhuri's formula, so that

$$\tilde{q} = \frac{1}{2}\left(1 + 3w_{ef}\right), \qquad \textit{where} \qquad w_{ef} = p_{ef}/\rho\,. \tag{9}$$

Then, $\tilde{q} < 0$ globally, if $w_{eff} < -1/3$ (e.g. dark energy).

Informed observer's approach

They include the relative-motion effect to the Raychaudhuri equation, so that

$$\tilde{q} = \frac{1}{2}\left[1 - \left(\frac{\lambda_T}{\lambda}\right)^2\right], \qquad \textit{where} \qquad \lambda_T^2 = \frac{2\vartheta}{9H}\,\lambda_H^2. \tag{10}$$

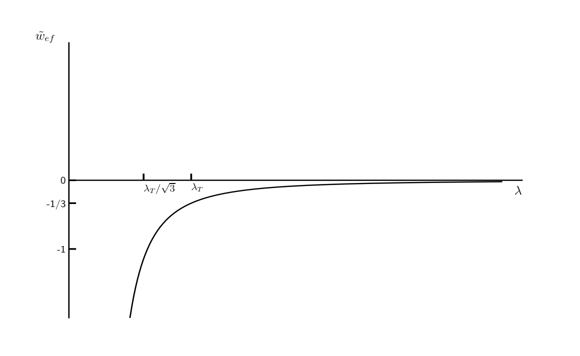
Then, $\tilde{q} < 0$ locally (i.e. when $\lambda < \lambda_T$).

Mimicking evolving dark energy: Deception No 3?

Apparent redshift evolution of w_{ef}

Combining (9) and (10) leads to the relation

$$w_{ef} = -\frac{1}{3}\left(\frac{\lambda_T}{\lambda}\right)^2, \quad \textit{so that} \quad w_{ef} \to \begin{cases} \simeq 0 \ \ \textit{when } \lambda \gg \lambda_T & (\textit{dust}) \\ = -1/3 \ \ \textit{when } \lambda = \lambda_T & (\textit{transition scale}) \\ = -1 \ \ \textit{when } \lambda = \lambda_T/\sqrt{3} & (\textit{phantom devide}) \\ < -1 \ \ \textit{when } \lambda < \lambda_T/\sqrt{3} & (\textit{phantom matter}) \end{cases} .$$



Universal acceleration as a relative-motion artefact

Uninformed observer's viewpoint

- Likely to misinterpret their slower local expansion as acceleration of the surrounding universe.
- Like passengers on a slowing down rain may believe that the trains next to theirs have accelerated away.
- Also likely to believe that the nature of the cosmic medium underwent a drastic change at the onset of the accelerated phase.

Informed observer's viewpoint

- To them, everything can be a deception caused by their bulk flow's local contraction.

Likelihood of living in a contracting bulk flow

- If there is no natural bias for expanding, or contracting, bulk flows on cosmological scales, the chances of residing in a contracting one should be close to 50%.
- Recent reconstruction of the local peculiar velocity field found it to have negative divergence (Pasten et al (2024)).

Observational signatures of the relative-motion effect

Theoretical predictions of the "tilted universe" scenario

- The observed $\tilde{q}$-profile, with $\tilde{q} \gtrless 0$ when $z \gtrless z_T$, follows naturally.
- The sky-distribution of $\tilde{q}$ should contain a Doppler-like dipole (trademark signature).
- The universe should appear to accelerate faster along a certain celestial direction and equally slower along the antipodal.
- The magnitude of the $\tilde{q}$-dipole should decrease with increasing scale/redshift.

Observational tests

- Over the last 15 years, several papers have reported a dipolar anisotropy in the universal acceleration (e.g. Cooke & Lynden-Bell, MNRAS (2010)).
- Wang & Wang, MNRAS (2014) reported a q-dipole consistent with a bulk flow of 270 km/sec on a scale of 100/h Mpc.
- Colin et al, A&A (2019) were the first to attribute the q-dipole to our peculiar motion (JLA).
- A dipolar anisotropy in the distribution of Ω_Λ, consistent with the tilted-universe scenario, was reported in Clocciatti et al, Ap. J. (2024).
- The q-dipole (in (Pantheon+) was found to decay with redshift in Sah et al, EPJC (2025).

Tilted cosmology scenario

Advocates that:

The recent cosmic acceleration can be an artifact of our peculiar motion

Main features

- No need for dark energy, or Λ.
- No need to modify GR.
- No need to abandon the FRW models.
- No coincidence problem.
- Physically motivated (peculiar flows do exist).
- Makes specific and distinct predictions.
- Can be tested against the observations.