Doppler-Like Dipoles in the Universal Expansion Due to Peculiar Flows

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Abstract We consider dipolar anisotropies in the universal expansion triggered by bulk peculiar flows. Starting with the deceleration parameter, we employ the familiar expansion tensor and introduce a new direction-dependent parameter, namely the deceleration tensor. Using linear cosmological perturbation theory we identify an apparent (Doppler-like) dipolar anisotropy in the sky distribution of the deceleration parameter, which is due to the observer's peculiar motion and it is closely analogous to the dipole seen in the Cosmic Microwave Background. In practice, this means that the bulk-flow observers should "see" their universe accelerating faster towards a certain point on the celestial sphere and equally slower towards the antipodal. Typically, the dipole axis should lie fairly close to that of the microwave background, though the two dipoles should not necessarily coincide. Also, given that peculiar velocities fade away on progressively larger scales, the magnitude of the dipolar anisotropy in the deceleration parameter should decrease with increasing redshift. Finally, we show that peculiar motions leave an apparent dipolar imprint in the sky distribution of the Hubble parameter as well.

1 Introduction

Relative motions have been known to introduce apparent, Doppler-like, dipolar anisotropies, triggered solely by the observer's motion towards, or away, from the source. In cosmology, the best known example is the dipole seen in the spectrum of the Cosmic Microwave Background (CMB). The latter is not believed to reflect a real generic anisotropy of the universe, but it has been treated as an artefact of our peculiar motion relative to the "smooth" universal expansion (Sciama 1967). In fact, according to the cosmological principle, every typical observer in the universe

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should see an analogous dipole, though not necessarily of the exact same magnitude and direction.[1] On these grounds. we may use the idealised CMB frame as the reference coordinate system of the universe, relative to which it makes sense to define and measure peculiar velocities.

Large-scale peculiar motions, also known as bulk flows (see Ade et al. (2014) and references therein), are believed to be the result of the ongoing structure-formation process. For this reason, peculiar velocities have been treated as a relatively recent (post recombination) addition to the kinematics of our universe. Theoretical studies, both Newtonian and relativistic, show that peculiar-velocity perturbations grow in time (Peebles 1980; Tsaprazi and Tsagas 2020). This means that the magnitude of any bulk velocity field should increase at later times (low redshifts–small scales) and fade away at higher redshifts. As a result, the relative-motion effects should become less pronounced as one moves away from the observer to progressively larger cosmological lengths.

Dipolar anisotropies in the sky-distribution of the deceleration parameter were first reported (to the best of our knowledge) about fifteen years ago by Cooke and Lynden-Bell (2010). Other reports followed (e.g. see Antoniou and Perivolaropoulos (2010); Wang and Wang (2014); Bengaly et al. (2015) and references therein), but it was not until recently that the q-dipole was attributed to our peculiar motion relative to the CMB frame by Colin et al. (2019). An analogous dipole, this time in the distribution of Ω_Λ was aslo recently reported in Clocchiatti et al. (2024). Although the issue is still open, if the aforementioned dipoles are confirmed, it could have pivotal implications for our understanding of the recent expansion history of the universe (perhaps even more–e.g. see Tsagas (2010, 2011); Secrest et al. (2021); Siewert et al. (2021); Krishnan et al. (2022); Singal (2022) for a fairly representative list of possibilities). In addition, there have also been recent claims of a dipolar anisotropy in the sky-distribution of the Hubble parameter, along an axis that is largely consistent with that of the aforementioned q-dipole (Migkas et al. 2020, 2021). Given the close relation between the Hubble and the deceleration parameters (the latter is essentially the time-derivative of the former), a dipolar anisotropy in one of them should (almost) necessarily ensure the same for the other.

The standard deceleration parameter (q) is a single scalar that monitors the average (isotropic) deceleration/acceleration rate of the universal expansion, which makes it unsuited for identifying anisotropies in the universal kinematics. One can circumvent the problem, by introducing three (scalar) deceleration parameters, one along each of the principal axis. Proceeding in this way and assuming a (perturbed) tilted Friedmann-Robertson-Walker (FRW) cosmology. it was demonstrated that bulk peculiar flows induce an apparent dipolar anisotropy in the deceleration parameter

[1] No real observer in the universe follows the CMB frame, but we all move relative to it. Our Milky Way and the Local Group of galaxies, for example, "drift" with peculiar velocity close to 600 km/sec (Kogut et al. 1993; Aghanim et al. 2020).

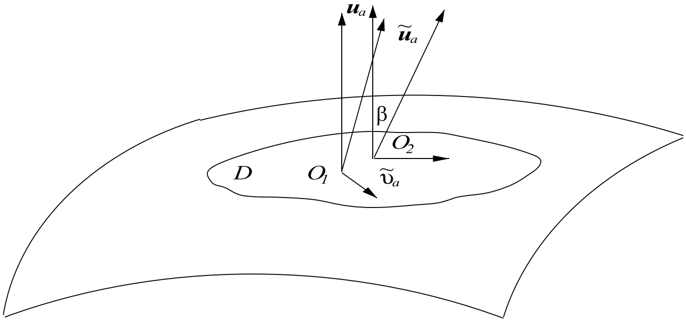

Fig. 1 Observers (O_1, O_2) with peculiar velocity $\tilde{v}_a$ relative to the reference u_a-field. The 4-velocities u_a and $\tilde{u}_a$ are related by the Lorentz boost $\tilde{u}_a = \tilde{\gamma}(u_a + \tilde{v}_a)$, where $\tilde{\gamma} = (1 - \tilde{v}^2)^{-1/2}$. The hyperbolic "tilt" angle β is defined so that $\cosh\beta = -u_a\tilde{u}^a = \tilde{\gamma}$, with $\tilde{\gamma} \simeq 1$ for non-relativistic peculiar motions (i.e. for $\tilde{v}^2 \ll 1$). Throughout this study the u_a-field is aligned with the idealised CMB frame, while $\tilde{u}_a$ is the 4-velocity of a real observers inside the bulk-flow D

along the observer's motion (Tsagas 2010, 2011). Here, we provide a more general way of incorporating directional dependencies, by introducing and using the deceleration tensor (Q_{ab}–see Sect. 3.1 below). The latter is a 3×3 spatial matrix, the trace of which reduces to the familiar scalar deceleration parameter, while its off-diagonal components monitor anisotropies in the rate of the universal deceleration/acceleration. These anisotropies can be genuine, namely features of the host spacetime, or apparent triggered by the observer's peculiar motion. In order to disentangle the two, it helps to employ a tilted cosmological model.[2]

Tilted cosmologies are best suited to the study of bulk peculiar flows, as they allow for two (at least) families of relatively moving observers. Typically, the first family is aligned with the idealised coordinate system of the CMB, while the second family are the real (the tilted) observers residing in galaxies like our Milky Way (see Fig. 1 in Sect. 2.1). Writing the deceleration tensor in the (reference) CMB frame and in the tilted coordinate system of the real observers and then comparing the two expressions, one can readily isolate the relative-motion effects in the components of the deceleration tensor. Among others, the comparison reveals an apparent Doppler like anisotropy in the deceleration parameter solely triggered by the observers' peculiar motion with respect to the CMB (see Sect. 3.3 here). More specifically, observers moving towards a certain point on the celestial sphere will assign a larger/smaller value to the deceleration parameter measured locally in the direction of their motion. Observers that move away from the same point in the sky, on the other hand, will assign an equally smaller/larger value to their local deceleration parameter. In other words, even when the tilted observers are surrounded by an isotropic distribution of sources (say of supernovae Ia), they will assign a more/less negative value to the

[2] Given the linear nature of the pursued analysis, our study applies to lengths larger than the anticipated homogeneity scale of the universe. The latter is typically set around the 100 Mpc threshold.

deceleration parameter measured along the direction of their peculiar motion and an equally less/more negative value along the antipodal.[3]

The close relation between the deceleration and the Hubble parameters ensures that that peculiar motions induce an exactly analogous apparent dipole in the sky-distribution of the Hubble parameter as well (see Sect. 3.4 here). We demonstrate this by working in a tilted almost-FRW cosmology and by means of the familiar expansion tensor. In addition to the analogies, however, there are differences as well, suggesting that neither the magnitudes nor the axes of these two kinematic dipoles should necessarily coincide.

2 Relative Motions in Cosmology

Observers living in typical galaxies move relative to the idealised reference frame of the universe. The latter has been typically identified with the coordinate system of the CMB, where the associated dipole vanishes by default. This makes the CMB frame the coordinate system, relative to which one should define and measure large-scale peculiar velocities.

2.1 The 4-Velocity Tilt

Consider two families of observers in an perturbed Friedmann universe. Let us also identify the former family with the "fictitious" observers that follow the CMB frame and the latter group with the "real" observers moving relative to it. When the peculiar motion is non-relativistic, the 4-velocities of the idealised and the real observers (u_a and $\tilde{u}_a$ respectively) are related by

$$\tilde{u}_a = u_a + \tilde{v}_a \,, \tag{1}$$

where $\tilde{v}_a$ represents the aforementioned peculiar velocity. Note that $\tilde{u}_a\tilde{u}^a = -1 = u_a u^a$, $u_a\tilde{v}^a = 0$ by construction and $\tilde{v}^2 = \tilde{v}_a\tilde{v}^a \ll 1$ at the non-relativistic limit (see also Fig. 1).

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[3] Additional anisotropies in the sky-distribution of the deceleration parameter may also appear, because the supernovae involved in the measurements are not isotropically distributed in space and they may also have individual peculiar motions, because the bulk flow is not spherically symmetric and the observer may not reside at its centre, because the peculiar motion may be generically anisotropic, etc (see also Sect. 3.3 later). Analysing all these different types of anisotropy, however, goes beyond the scope of this study. Here, instead, we are focusing on the trademark signature of peculiar flows, that is on an apparent (Doppler-like) dipolar anisotropy in the spectrum of the deceleration parameter, solely triggered by the observers' motion relative to the distant sources.

Introducing two 4-velocity fields into the spacetime implies defining two temporal directions, along u_a and $\tilde{u}_a$ respectively, as well as an equal number of 3-spaces orthogonal to these 4-velocity vectors. Then, the metric (g_{ab}) of the host spacetime decomposes as $g_{ab} = h_{ab} + u_a u_b = \tilde{h}_{ab} + \tilde{u}_a \tilde{u}_b$, with h_{ab} and $\tilde{h}_{ab}$ projecting orthogonal to their corresponding 4-velocities (i.e. $h_{ab}u^b = 0 = \tilde{h}_{ab}\tilde{u}^b$). Moreover, the operators $^{\cdot} = u^a\nabla_a$ and $' = \tilde{u}^a\nabla_a$ define time derivatives along the u_a and the $\tilde{u}_a$ fields respectively, while the associated spatial gradients are $\mathrm{D}_a = h_a{}^b\nabla_b$ and $\tilde{\mathrm{D}}_a = \tilde{h}_a{}^b\nabla_b$. Overall, by introducing the u_a and the $\tilde{u}_a$ fields, we have achieved a "double" 1+3 splitting of the spacetime into time and 3-D space (Tsagas et al. 2008; Ellis et al. 2012).

One is free to choose any of these two groups of observers to monitor the implications of large-scale peculiar motions for the kinematics of the universal expansion. Here, we will adopt the viewpoint of the real (the "tilted") observes, namely those living in typical galaxies like ours, instead of following their fictitious CMB counterparts.[4] Technically speaking, this means that the whole of our analysis will take place in the "tilted" frame of the bulk peculiar motion.[5]

2.2 *Tilted Kinematics*

The kinematic details of the three velocity fields involved in Eq. (1) follow after splitting their gradients into their irreducible components. More specifically, we have

$$\nabla_b u_a = \frac{1}{3}\,\Theta h_{ab} + \sigma_{ab} + \omega_{ab} - A_a u_b \qquad \text{and} \qquad \nabla_b \tilde{u}_a = \frac{1}{3}\,\tilde{\Theta}\tilde{h}_{ab} + \tilde{\sigma}_{ab} + \tilde{\omega}_{ab} - \tilde{A}_a\tilde{u}_b\,, \tag{2}$$

for the two 4-velocities and

$$\tilde{\mathrm{D}}_b\tilde{v}_a = \frac{1}{3}\,\tilde{\vartheta}\tilde{h}_{ab} + \tilde{\varsigma}_{ab} + \tilde{\varpi}_{ab}\,, \tag{3}$$

for the peculiar velocity (Tsagas et al. 2008; Ellis et al. 2012). In the above, $\Theta = \mathrm{D}^a u_a$ is the volume scalar monitoring the expansion/conraction of the associated family of observers (when positive/negative). In addition, $\sigma_{ab} = \mathrm{D}_{\langle b}u_{a\rangle}$ and $\omega_{ab} = \mathrm{D}_{[b}u_{a]}$

[4] Hereafter, we will use the term "tilted" when referring to the real observers, because of the hyperbolic "tilt" angle (β) formed between their 4-velocity ($\tilde{u}_a$) and that of the reference u_a-field (see Fig. 1).

[5] One may also take the view point of the idealised observers and pursue the analysis in the coordinate system of the CMB, assuming that the latter moves with "peculiar" velocity $v_a = -\tilde{v}_a$ relative to the tilted frame. It goes without saying (and it is also straightforward to show) that the results are physically identical.

are the shear and the vorticity tensors respectively.[6] The former describes shape distortions in the associated worldline congruence and the latter its rotation. Finally, $A_a = u^b\nabla_b u_a$ is the 4-acceleration vector, implying non-geodesic motion. Clearly, exactly analogous definitions hold for the "tilded" variables on the right-hand side of (2b). Similarly, expression (3) provides the irreducible decomposition of the peculiar-velocity field, as seen by the real observers. Hence, $\tilde{\vartheta} = \tilde{\mathrm{D}}^a\tilde{v}_a$, $\tilde{\varsigma}_{ab} = \tilde{\mathrm{D}}_{\langle b}\tilde{v}_{a\rangle}$ and $\tilde{\varpi}_{ab} = \tilde{\mathrm{D}}_{[b}\tilde{v}_{a]}$ are respectively the volume scalar, the shear and the vorticity tensors of the bulk peculiar flow.

The kinematic variables defined in Eqs. (2) and (3) are related with each other and the relations depend on the magnitude of the peculiar velocity and on the symmetries of the host spacetime (see Maartens (1998) for the full nonlinear expressions). Assuming non-relativistic peculiar motions in a perturbed (almost-FRW) universe, leads to the linear relations

$$\tilde{\Theta} = \Theta + \tilde{\vartheta}\,, \qquad \tilde{\sigma}_{ab} = \sigma_{ab} + \tilde{\varsigma}_{ab}\,, \qquad \tilde{\omega}_{ab} = \omega_{ab} + \tilde{\varpi}_{ab} \tag{4}$$

and

$$\tilde{A}_a = A_a + \tilde{v}_a' + H\tilde{v}_a\,, \tag{5}$$

with $H = \Theta/3$ representing the background Hubble parameter. Consequently, the kinematics of the two frames differ and the differences are entirely due to relative-motion effects. On these grounds, the aforementioned effects should be local, though the affected scales can be large enough (e.g. of the order of few hundred Mpc) to acquire cosmological relevance.

3 Peculiar Motions and the Deceleration Parameter

Relative motions are typically associated with apparent (Doppler-like) dipolar anisotropies triggered by the observer's peculiar flow. The CMB dipole, for example, has been attributed to the peculiar motion of our Local Group. In order to identify similar signatures in the deceleration/acceleration rate of the universe, it helps to introduce an associated deceleration tensor.

[6] Angled brackets denote the symmetric, traceless component of second-rank tensors, while square ones indicate their antisymmetric part. Put another way, $\sigma_{ab} = (\mathrm{D}_b u_a + \mathrm{D}_a u_b)/2 - (\mathrm{D}^c u_c/3)h_{ab}$ and $\omega_{ab} = (\mathrm{D}_b u_a - \mathrm{D}_a u_b)/2$.

3.1 The Deceleration Tensor

The standard deceleration parameter (q) is a scalar variable that monitors the mean deceleration/acceleration of the universal expansion. The latter is not fully isotropic, however, primarily due to effects triggered by structure formation. Here, we will describe anisotropies in the global deceleration/acceleration of the universe by introducing the deceleration tensor

$$Q_{ab} = -\left(h_{ab} + \frac{9}{\Theta^2}\, h_a{}^c h_b{}^d \dot{\Theta}_{cd}\right), \tag{6}$$

where

$$\Theta_{ab} = \frac{1}{3}\,\Theta h_{ab} + \sigma_{ab} + \omega_{ab}\,, \tag{7}$$

is the more familiar expansion tensor (e.g. see Tsagas et al. (2008)).[7] By construction, both Θ_{ab} and Q_{ab} are spacelike tensors (i.e. $\Theta_{ab}u^a = 0 = \Theta_{ab}u^b$ and $Q_{ab}u^a = 0 = Q_{ab}u^b$), with a symmetric as well as an antisymmetric part.[8] In addition, as expected, the traces (6) and (7) give

$$Q = 3q \qquad \text{and} \qquad \Theta = 3H\,, \tag{8}$$

where $q = -[1 + (3\dot{\Theta}/\Theta^2)]$ and $H = \Theta/3$ are the familiar deceleration and Hubble parameters respectively (Tsagas et al. 2008). In other words, the deceleration tensor is a 3×3 matrix with its diagonal components measuring the mean deceleration/acceleration of the expansion. The non-diagonal components of Q_{ab}, on the other hand, describe anisotropies due to shear-like distortions and rotational perturbations. Exactly analogous is the role of the expansion tensor.

Suppose that n_a is the unit vector along a given spatial direction, so that $n_a n^a = 1$ and $u_a n^a = 0$. Then, the deceleration/acceleration of the expansion measured along n_a is given by the (twice contracted) scalar $Q_{ab}n^a n^b$. On using definition (6), the latter reads

$$Q_{ab}n^a n^b = q - \frac{9}{\Theta^2}\,\dot{\sigma}_{ab}n^a n^b\,, \tag{9}$$

since $\dot{\omega}_{ab}n^a n^b = 0$ and $\dot{h}_{ab}n^a n^b = (A_a u_b + u_a A_b)n^a n^b = 0$. Therefore, the deceleration parameter measured in a given spatial direction consists of its mean value plus/minus corrections due to shear-like anisotropies. These are triggered by temporal variations in the effective shear distribution along the direction in question.

[7] Typically, the expansion tensor does not include rotational effects. Nevertheless, we have incorporated the vorticity tensor into definition (7) for completeness, although rotation will play no direct role in our analysis.

[8] Isolating the symmetric traceless component and the antisymmetric part of (6), gives $Q_{\langle ab\rangle} = -9h_a{}^c h_b{}^d \dot{\sigma}_{cd}/\Theta^2$ and $Q_{[ab]} = -9h_a{}^c h_b{}^d \dot{\omega}_{cd}/\Theta^2$ respectively. Also, it goes without saying that $\Theta_{\langle ab\rangle} = \sigma_{ab}$ and $\Theta_{[ab]} = \omega_{ab}$.

Finally, applied to an FRW universe, where $\Theta = 3H$ and $\sigma_{ab} = 0 = \omega_{ab}$ by default, expressions (6) and (9) reduce to

$$Q_{ab} = qh_{ab} \qquad \text{and} \qquad Q_{ab}n^a n^b = q\,, \tag{10}$$

as expected given the spatial isotropy of the Friedmann models.

3.2 The Deceleration Tensor in Tilted Universes

Let us consider a tilted almost-FRW universe and allow for a group of real observers living in a typical galaxy like our Milky Way and moving relative to the CMB frame with 4-velocity $\tilde{u}_a$ and bulk peculiar velocity $\tilde{v}_a$ (see Fig. 1 in Sect. 2.1 earlier). Written in the tilted frame, the deceleration tensor defined in (6) reads

$$\tilde{Q}_{ab} = -\left(\tilde{h}_{ab} + \frac{9}{\tilde{\Theta}^2}\,\tilde{h}_a{}^c\tilde{h}_b{}^d\tilde{\Theta}'_{cd}\right), \tag{11}$$

with $\tilde{\Theta}_{ab} = (\tilde{\Theta}/3)\tilde{h}_{ab} + \tilde{\sigma}_{ab} + \tilde{\omega}_{ab}$ and the primes denoting time differentiation along the $\tilde{u}_a$-field.[9]

Starting form (11), substituting relations (4), employing definitions (6) and (7), while keeping up to first-order terms, we obtain

$$\tilde{Q}_{ab} - Q_{ab} = 2qu_{(a}\tilde{v}_{b)} - \frac{1}{H^2}\left(\frac{1}{3}\,\tilde{\vartheta}'h_{ab} + \varsigma'_{ab} + \tilde{\varpi}'_{ab}\right), \tag{12}$$

given that $\tilde{\Theta} = \Theta = 3H$ in the FRW background and $\tilde{h}_{ab} = h_{ab} + 2u_{(a}\tilde{v}_{b)}$ at the linear level. The above applies to a perturbed Friedmann universe and relates the deceleration tensor ($\tilde{Q}_{ab}$) measured in the tilted frame of the real observers to the one in the CMB frame of their fictitious counterparts (Q_{ab}). As expected, in the absence of peculiar motions, the two tensors coincide.

Taking the trace of (12), using (8), recalling that $\tilde{\vartheta}/H \ll 1$ during the linear regime, as well as keeping in mind that $h_a{}^a = 3$ and $u_a\tilde{v}^a = 0 = \varsigma'_a{}^a = \tilde{\varpi}'_a{}^a$ always, provides the following linear relation between the deceleration parameters in the aforementioned two coordinate systems

$$\tilde{q} - q = \frac{\tilde{\vartheta}'}{3\dot{H}}\left(1 + \frac{1}{2}\,\Omega\right) = -\frac{\tilde{\vartheta}'}{3H^2}\,, \tag{13}$$

[9] Alternatively, one can take the viewpoint of the idealised CMB observer, by accounting for their "peculiar flow" relative to the tilted frame with velocity v_a (where $v_a = -\tilde{v}_a$ at the non-relativistic limit). It goes without saying that the change of frames leaves the results and the conclusions unaffected and physically identical.

with $\Omega = \rho/3H^2$ representing the density parameter of the FRW background. Hence, q and $\tilde{q}$ differ and their difference depends on the dimensionless ratio $\tilde{\vartheta}'/\dot{H}$ (or equivalently on $\tilde{\vartheta}'/H^2$). The above is in complete agreement with Tsagas (2011) and Tsaprazi and Tsagas (2020), where the interested reader is referred to for a discussion on the relative-motion effects on the (scalar) deceleration parameter. Here we note that, whereas $\tilde{\vartheta}/H \ll 1$ at the linear perturbative level, the ratios $\tilde{\vartheta}'/\dot{H}$ and $\tilde{\vartheta}'/H^2$ are not necessarily small and their impact increases with decreasing scale. This allows for an apparent change in the sign of the deceleration parameter (from positive to negative) measured locally by observers inside slightly contracting bulk flows (Tsagas 2010, 2011, 2021, 2022; Tsagas and Kadiltzoglou 2015).

3.3 Doppler-Like Dipole in the Deceleration Parameter

Projecting expression (12) along a given spatial direction (determined by the unit vector n_a, with $u_a n^a = 0$), using definitions (6) and (11) for Q_{ab} and $\tilde{Q}_{ab}$ respectively, while keeping in mind that $(\tilde{\vartheta}/3)\tilde{h}_{ab} + \tilde{\varsigma}_{ab} = \tilde{\mathrm{D}}_{(b}\tilde{v}_{a)}$, leads to

$$\tilde{Q}_{ab}n^a n^b - Q_{ab}n^a n^b = -\frac{1}{H^2}\left(\tilde{\mathrm{D}}_b\tilde{v}_a\right)' n^a n^b \,, \tag{14}$$

to first approximation. Moreover, by means of the linear commutation law $\left(\tilde{\mathrm{D}}_b\tilde{v}_a\right)' = \tilde{\mathrm{D}}_b\tilde{v}'_a - H\tilde{\mathrm{D}}_b\tilde{v}_a$, we arrive at

$$\begin{aligned}\tilde{Q}_{ab}n^a n^b - Q_{ab}n^a n^b &= \frac{1}{H}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}_b n^b\right) - \frac{1}{H^2}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}'_b n^b\right) \\ &= \frac{1}{H}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right) - \frac{1}{H^2}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}'\cos\psi\right),\end{aligned} \tag{15}$$

In the above $\tilde{v}$ and $\tilde{v}'$ are the magnitudes of the two vectors, while ϕ and ψ represent the trigonometric angles between $\tilde{v}_a$, $\tilde{v}'_a$ and n_a respectively (with $0 \le \phi,\ \psi \le \pi$). The right-hand side of (15) ensures that there are differences between the directional measurements of the deceleration parameter made in the two frames (i.e. that $\tilde{Q}_{ab}n^a n^b \neq Q_{ab}n^a n^b$), triggered solely by the peculiar flow of the tilted (the real) observer. More specifically, unless the two terms on the right-hand side of the above "conspire" to cancel each other out, they induce an apparent (Doppler-like) dipolar anisotropy in the sky distribution of the deceleration parameter along the observer's peculiar flow.

In order to demonstrate this claim, let us take the standard approach that the universe appears isotropic to observers following the idealised CMB frame. Practically,

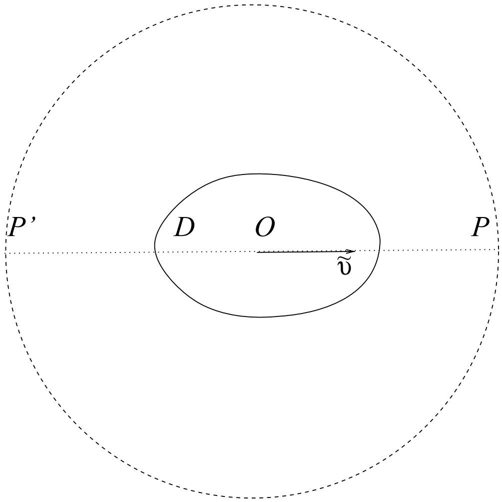

Fig. 2 Consider an isotropic sky-distribution of identical distant (comoving) sources (e.g. of supernovae I a–circular dashed line) around the observer (O), living in a typical galaxy inside a bulk flow (D) and moving with peculiar velocity $\tilde{v}$ relative to the sources. Assuming that the observer "approaches" point P on the celestial sphere and simultaneously "moves away" from the antipodal point P', they will "see" an apparent (Doppler-like) dipole in the sky-distribution of the deceleration parameter forming along the direction of their peculiar motion (dotted line). Note that we ignore any side effects due to the potential anisotropy of the bulk flow itself

this means setting $Q_{ab}n^an^b = q$ in Eq. (15)–see also relation (10a) in Sect. 3.1 earlier. Then, assuming (for simplicity) that the first term on the right-hand side of (15) dominates, the latter expression reduces to

$$\tilde{Q}_{ab}n^an^b = q + \frac{1}{H}\, n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right)\,, \tag{16}$$

Therefore, keeping the direction vector (n_a) fixed, we arrive at

$$\tilde{Q}_{ab}n^an^b = q + \frac{1}{H}\, n^a\tilde{\mathrm{D}}_a\tilde{v}\,, \qquad \text{and} \qquad \tilde{Q}_{ab}n^an^b = q - \frac{1}{H}\, n^a\tilde{\mathrm{D}}_a\tilde{v} \tag{17}$$

when $\tilde{v}_a \uparrow\uparrow n^a$ and $\tilde{v}_a \uparrow\downarrow n^a$ respectively (alternatively for $\phi = 0$ and $\phi = \pi$). Consequently, depending on whether the directional gradient $n^a\tilde{\mathrm{D}}_a\tilde{v}$ is positive/negative, observers moving towards a certain point on the sky will assign an increased/decreased value to the deceleration parameter in that direction, compared to the value measured in the CMB frame. In contrast, observers moving away from the aforementioned point will assign an equally decreased/increased value to $\tilde{Q}_{ab}n^an^b$. Accordingly, observers "approaching" a certain point on the celestial sphere will measure an increased/decreased value for their deceleration parameter in the direction of that point and an equally decreased/increased value towards the antipodal point they are moving away from.[10] Therefore, the bulk-flow observers should "measure" faster acceleration along one celestial direction and equally slower in the opposite. Then, the sky distribution of the deceleration parameter should exhibit an apparent (Doppler-like) dipole entirely due to their peculiar motion (e.g. see Fig. 2). This distinct type of anisotropy is the "trademark signature" of relative motion and fairly straightforward to disentangle from anisotropies of different origin, like those due to structure formation for example. Analysing the Pantheon+ data set and those of the upcoming LSST-DA survey, for example, should provide the opportunity to do so.

According to Eq. (17), the magnitude of the induced dipole is determined by the value of the projected gradient $n^a\tilde{\mathrm{D}}_a\tilde{v}$. Unfortunately, the current bulk-flow surveys provide estimates of the mean bulk velocity only and not of its spatial gradients. This makes it impossible to estimate quantitatively the magnitude of the aforementioned dipole. Nevertheless, the gradients seen on the right-hand sides of (17a) and (17b) ensure that the induced dipole should appear stronger on smaller scales (i.e. closer to the observer) and weaker on progressively larger lengths (i.e. away from the observer). This is also corroborated by the fact that the peculiar velocities themselves also fade away with scale/redisift.

In general, the induced anisotropy depends on the angle (ψ) between $\tilde{v}'_a$ and n_a as well (see Eq. (15) above), in which case expression (16) generalises to

$$\tilde{Q}_{ab}n^an^b = q + \frac{1}{H}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right) - \frac{1}{H^2}\,n^a\tilde{\mathrm{D}}_a\left(\tilde{v}'\cos\psi\right)\,. \tag{18}$$

The last term on the right-hand side of the above also triggers an apparent (Doppler-like) dipolar anisotropy in the sky distribution of the deceleration parameter, though this time the axis is not necessarily collinear with the peculiar velocity vector ($\tilde{v}_a$). Clearly, the overall q-dipole is the linear combination of the aforementioned two. In practice, this means that the dipole axis in the q-distribution should not necessarily coincide with that of the CMB dipole, assuming that the latter is purely kinematical and therefore collinear to $\tilde{v}_a$.

In all of the above, the observer has been residing near the centre of an almost spherical bulk flow, which is moving coherently relative to a largely isotropic distribution of distant sources. Violating any of these assumptions, should inevitably lead to additional types of anisotropy on top of the dipolar. For instance, observers residing in the periphery of the dark flow will see a hemisphere anisotropy in the sky-distribution of the deceleration parameter. Anisotropies in the bulk-flow kinematics are likely to cause quadruple anisotropies in the observed q-distribution, the

[10] Consider an isotropic distribution of distant sources, say of supernovae type Ia, around the bulk-flow observers, whose peculiar motion "brings them closer" to a certain source on the celestial sphere, while it "pushes them away" from its identical counterpart at the antipodal. Then, despite the isotropy of the sources, the moving observers will "see" an apparent (Doppler-like) dipole forming in the sky distribution of the deceleration parameter along the direction of their peculiar motion (qualitatively analogous to that seen in the CMB spectrum–see Fig. 2).

individual peculiar motions of the sources could further complicate the overall picture, and so on. Nevertheless, of all the different types of anisotropy, the dipolar one is the "trademark" signature of relative motion and the reason we have focused on it.

3.4 Doppler-Like Dipole in the Hubble Parameter

Given the close relation between the deceleration and the Hubble parameters, one expects to see peculiar motions inducing an analogous apparent (Doppler-like) dipole in the sky-distribution of the latter. In general, anisotropic expansion is monitored by the expansion tensor, defined in Eq. (7) of Sect. 3.1. Employing Θ_{ab}, together with its tilted counterpart ($\tilde{\Theta}_{ab}$–see expression (11) in Sect. 3.2) and the linear relations (4), gives

$$\tilde{\Theta}_{ab}n^an^b - \Theta_{ab}n^an^b = n^b\tilde{\mathrm{D}}_b\left(\tilde{v}_an^a\right) = n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right)\,, \tag{19}$$

with $0 \leq \phi \leq \pi$. The above ensures that the rate of the Hubble expansion measured along a spatial direction n_a by the tilted observer, differs from the one measured by their CMB counterparts, entirely due their relative motion. More specifically, the peculiar flow of the tilted frame triggers a Doppler-like anisotropy in the sky distribution of the Hubble parameter, which depends on the direction of the observer's motion and it is exactly analogous to that seen in the deceleration parameter (see previous section). Indeed, following (19), we have

$$\tilde{\Theta}_{ab}n^an^b - \Theta_{ab}n^an^b = n^a\tilde{\mathrm{D}}_a\tilde{v} \qquad \text{and} \qquad \tilde{\Theta}_{ab}n^an^b - \Theta_{ab}n^an^b = -n^a\tilde{\mathrm{D}}_a\tilde{v} \tag{20}$$

when $\tilde{v}_a \uparrow\uparrow n^a$ and $\tilde{v}_a \uparrow\downarrow n^a$ respectively (alternatively for $\phi = 0$ and for $\phi = \pi$). Given that in the idealised CMB frame the universal expansion appears isotropic, we may set $\Theta_{ab}n^an^b = H$, in which case the above relations combine to

$$\tilde{\Theta}_{ab}n^an^b = H \pm n^a\tilde{\mathrm{D}}_a\tilde{v}\,. \tag{21}$$

Accordingly, depending on the sign of peculiar-velocity gradient, the bulk-flow observers will assign an increased/decreased value to the Hubble parameter measured along their motion and an equally decreased/increased value in the opposite direction they are moving away from. Clearly the resulting dipolar anisotropy is not real, but an apparent effect triggered solely by the observers' peculiar motion relative to the CMB frame (see also Fig. 2 in Sect. 3.3 previously).[11]

[11] As in the case of the deceleration parameter discussed previously, we have assumed that the astrophysical sources used to measure the Hubble parameter are isotropically distributed in space and they are simply comoving with the universal expansion. We have also situated the observer near the centre of an almost spherical bulk flow and ignored any anisotropies in the kinematics of the peculiar flow, as well as the peculiar motions of the sources.

As in the case of the deceleration parameter discussed before, we need (the still unavailable) data for the gradient of the peculiar-velocity field to estimate the magnitude of the H-dipole. On the other hand, the presence of the aforementioned gradient in Eq. (21) ensures that the magnitude of the induced dipole should be stronger on relatively smaller scales and it should decay away on progressively higher redshifts, where both the peculiar-velocity field and its gradient are expected to decay. Recall that the same also applies to the apparent dipole induced to the deceleration parameter by the observers' peculiar motion (see previous section). The latter, however, has additional effects coming from the gradient of the "peculiar acceleration" (see Eq. (18) in Sect. 3.3). This is to be expected, since the deceleration parameter is essentially the time derivative of the Hubble parameter. Therefore, in principle the two dipoles should differ (to a larger or lesser degree) both in magnitude and in direction.

4 Discussion

We are all familiar from everyday life with the Doppler effect, which typically manifests itself as an apparent shift in the frequency of a sound signal, in response to our motion towards the source, or away from it. The frequency appears to increase as we approach the source and to decrease, by an equal amount, as we recede from it. It goes without saying that the same also happens when the source moves towards, or away, from us. Therefore, observers moving relative to an isotropic distribution of identical sources, will detect an apparent dipolar anisotropy in the frequency along the direction of their motion. More specifically, the signal will appear to shift to higher frequencies towards the direction of the motion and to drop to equally lower frequencies towards the antipodal. Clearly, such an anisotropy is not real but a mere relative-motion effect.

In cosmology, the most celebrated Doppler-like anisotropy is the temperature dipole seen in the CMB spectrum, which is not treated as a manifestation of some underlying anisotropy of the universe. Instead, the standard and most straightforward interpretation is that the observed dipole is a Doppler-like effect, triggered by our peculiar motion relative the highly isotropic distribution of the CMB photons. Based on that, the peculiar velocity of the Local Group of galaxies has been estimated close to 600 km/sec. Moreover, provided the Cosmological Principle holds, every observer in the universe should see an analogous dipole in their CMB spectrum, though not necessarily of the exact same magnitude and direction. This makes the coordinate system where the CMB dipole vanishes, the reference frame of the universe.

Dipolar anisotropies are also the "trademark" signature of relative motion with respect to an isotropic distribution of sources. One therefore expects to find (apparent) dipoles when observing distant astrophysical sources, which are distributed more or less randomly and are far enough to simply follow the universal expansion. At present, the sources used to measure the Hubble and the deceleration parameters satisfy these requirements, at least to some degree. It should not come as a surprise therefore, that reports have appeared in the literature claiming the presence of dipole anisotropies in the sky-distribution of both of these cosmological parameters. This work provides a simple theoretical explanation to the dipoles reported in Cooke and Lynden-Bell (2010), Colin et al. (2019) and Migkas et al. (2021), as apparent Doppler-like effects triggered by our peculiar flow with respect to the CMB expansion. In so doing, we have employed linear cosmological perturbation theory and isolated any other type of anisotropy. These may include, for example, hemisphere anisotropies in the sky-dietribution of the deceleration parameter due to the observers position away for the bulk-flow center, quadruple anisorropies in the observed q-distribution caused by anisotropies in the bulk-flow kinematics, and so on (see Sect. 3.3 earlier and also footnote 2 in Sect. 1).

Our starting point was a tilted almost-FRW cosmology, equipped with two families of relatively moving observers. These are the fictitious observers following the idealised CMB frame and their real counterparts following the tilted frame and living in galaxies like our Milky Way (see Fig. 1 in Sect. 2.1). Comparing the deceleration and the Hubble parameters between the two frames, enabled us to distinguish the real effects from the apparent ones, namely from those caused by the observers' peculiar flow. To establish the relative-motion effects on the deceleration parameter, we defined and introduced the deceleration tensor, which extends its scalar counterpart and it can naturally incorporate all types of directional anisotropies (see definition (6) in Sect. 3.1). Our results showed that observers moving relative to the CMB frame, will assign higher/lower values to the deceleration parameter measured along their direction of motion and equally lower/higher values towards the antipodal. This will result into an apparent dipolar anisotropy in the sky distribution of the measured deceleration parameter, solely triggered by relative-motion effects. The magnitude of the induced dipole was found to depend on the gradient of the peculiar velocity ($\tilde{v}$), as well as on that of the peculiar acceleration ($\tilde{v}'$), both along the direction of the observers' motion (see Eq. (18) in Sect. 3.3). Given that, we expect the strength of the apparent dipole to weaken on progressively larger scales (i.e. at higher redshifts). Also, the axis of the q-dipole does not need to coincide with that of its CMB counterpart.

The same principles also lead to an apparent dipolar anisotropy in the sky-distribution of the Hubble parameter. This is to be expected, given the close relation between the two cosmological parameters (one is the time-derivative of the other). Assuming that the distant sources used in the measurements of the Hubble value are isotropically distributed in space, observers moving relative to these sources will "see" an apparent dipole forming along their direction of motion. This time, however, the decisive factor is only the gradient of the peculiar velocity (see Eq. (21) in Sect. 3.4), which implies that the deceleration and the Hubble dipoles should differ (to a lesser or larger degree) both in their magnitudes and in their directions.

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